

CO³ TRAFFIC DEMAND, DELAY, AND USER COST MODEL¹

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ABSTRACT: The Construction Congestion Cost System (CO³) is an integrated set of tools to estimate the impact of traffic maintenance contract provisions on congestion, road user cost, and construction cost. We use CO³ to produce realistic budgets and select practical contracting methods that provide an acceptable balance between construction cost and congestion. This paper describes the underlying CO³ model of traffic demand, delay, and user cost, particularly the methods by which CO³ calculates traffic backup and delay, diverted and cancelled trips, and road user cost. It provides examples that illustrate the CO³ model and methods. CO³ models the common characteristic of construction work zones by which road capacity can vary from hour to hour as lanes close and open and work conditions change. Traffic demand also varies with time of day, and drivers may divert to an alternate route or cancel trips because of delays caused by construction. CO³ estimates traffic delay due to traffic congestion as a function of demand and capacity and it estimates traffic cancellations and diversions as functions of traffic delay.

1 INTRODUCTION

The Construction Congestion Cost System (CO³) is an integrated set of tools to estimate the impact of traffic maintenance contract provisions on congestion, road user cost, and construction cost (Carr 1997). With it we estimate and calculate traffic input variables, traffic delay and user cost impact, construction cost impact, integrated user cost and construction cost impact, period costs for contract provisions, and traffic related contract payments. We use CO³ to produce realistic budgets and select practical contracting methods that provide an acceptable balance between construction cost and congestion. We compare user cost and construction cost of alternative contract provisions to help us select the best project alternative. Therefore, CO³ helps us select among alternative methods of maintaining traffic during construction, and it helps us select contract period costs for contract provisions that provide incentives for reducing congestion impacts during construction. This paper describes only the underlying CO³ model of traffic demand, delay, and user cost, particularly the methods by which it calculates traffic backup and delay, diverted and cancelled trips, and user cost.

CO³ models the common characteristic of construction work zones by which road capacity can vary from hour to hour as lanes close and open and work conditions change. Traffic demand also varies with time of day, and traffic backs up when demand exceeds capacity. Some motorists divert to an alternate route or cancel trips because of delays caused by construction. Road user cost results from traffic delays, from increased travel time and distance of alternate routes, and from trip cancellations caused by congestion. CO³ calculates backups, delays, diverted traffic, and can-

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celled traffic to measure congestion. It calculates user cost as a measure of congestion that is integrated with construction cost in selecting among alternative contract provisions.

CO³ models the circular relationship between work zone demand and delay, in which diversions and cancellations are functions of delay, work zone demand is a function of diversions and cancellations, and delay is a function of work zone demand. It models traffic as an integrated stream of two classes of vehicles, trucks and cars, that share backups but differ in diversion and cancellation sensitivity. It models and estimates traffic delay due to traffic congestion as a function of demand and capacity and it estimates traffic cancellations and diversions as functions of traffic delay. It calculates traffic backup that is due to actual demand on the work zone exceeding work zone capacity.

The CO³ system is implemented in a Microsoft Excel spreadsheet. The spreadsheet and CO³ User Manual can be downloaded from the worldwide web at <http://grader.engin.umich.edu/>. CO³ was developed with financial support from the Michigan Department of Transportation to allow it to reduce the impact of highway construction on Michigan travelers.

2 BACKGROUND

An analytical procedure to estimate traffic delay and congestion and assess the tradeoffs in cost-effectiveness of alternative measures was described in (“Alternative” 1978). This was implemented in a microcomputer spreadsheet that is still widely used (Morales 1986). The underlying model includes only delay and user cost of vehicles in the queue formed by demand exceeding capacity. The QUEWZ model provides a recent model of traffic flow through lane closures in freeway work zones and queue lengths and road user costs that result (Krammes, Ullman, Memmott, and Dudek 1993). QUEWZ was designed for Texas urban freeways with continuous frontage roads, and it diverts all traffic that would cause a queue greater than a specified maximum length or motorist delay greater than a specified maximum delay. Microscopic traffic simulation models can also be applied to freeway incident management (Cragg and Demetsky 1995). However, this is generally impractical for broad application to work zone analysis for incentive contracts, particularly for project scope and budget decisions.

The FHWA has provided for incentive/disincentive and related clauses for highway construction based on road user costs since 1984 (“Incentive/disincentive” 1989), and practicalities of such clauses were described by Riley (1984), and Newman and Heijl (1984). Construction work zones can be considered a special case of freeway incident management in which interruptions of normal traffic is planned and controlled, and much can be learned from freeway incident management literature (Reiss 1991). Traffic/congestion management recommendations are summarized in (“Traffic” 1992), and alternative contract management techniques are described in (“Contract” 1997). Incentive-based highway contracting methods of various forms have been widely implemented with significant success (Jaraiedi, Plummer, and Aber 1995) (Herbsman, Chen, and Epstein 1995) (Herbsman 1995). The FHWA 1998 National Strategic Plan includes initiatives to stabilize congestion and reduce delays caused by construction (“Federal” 1997).

3 DEMAND, ROUTES, AND DELAY

3.1 DESIGN DEMAND

Design demand (Vph) = number of vehicles that travel through the site under normal conditions when it is not a construction site. Design demand is generally based on historical demand = sample(s) of number of vehicles per hour that traveled through the site under normal conditions at

the most recent sampling. Design demand can be estimated directly, but it normally will be directly calculated from **historical demand**, with adjustment for **years of growth** between the year in which historical demand is determined and the year in which the work will be performed, based on estimated **annual traffic growth** using input shown in Fig. 1. CO³ calculates design demand using the following equation:

$$\text{design demand} = (\text{historical demand}) * [1 + (\text{annual traffic growth})]^{(\text{years of growth})} \dots\dots\dots (1)$$

For example, using values in Fig. 1, if a period's historical demand = 3,124 Vph, the period's design demand is

$$\text{design demand} = (3,124) * (1 + 0.03)^2 = 3,314 \text{ Vph}$$

These values of historical demand and design demand are shown for 8:00 am to 9:00 am (8A) in Table 1, which also includes values for 9:00 am to 2:00 pm. [Note: For brevity, the example in this paper is for six periods, as shown in Table 1, rather than the 24 periods of CO³ software.]

period length (min)	60	
annual traffic growth (%)	3.00%	
years of growth	2	
VEHICLE INPUT	cars	trucks
design demand (%)	90.0%	10.0%

Fig. 1 - Input of Traffic and User Cost Constants³

Table 1. Period Capacity and Demand

period (hr)	capacity (V/hr)	historical demand (V/hr)	design demand (V/hr)	total period decrease (V/hr)	car actual demand (V/hr)	truck actual demand (V/hr)	actual demand (V/hr)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
8 A	3,400	3,124	3,314	0	2,983	331	3,314
9 A	1,400	2,436	2,584	571	1,773	240	2,013
10 A	1,400	2,051	2,176	810	1,177	188	1,366
11 A	1,400	1,436	1,523	431	955	137	1,092
12 P	1,400	1,513	1,605	282	1,171	152	1,323
1 P	3,400	2,099	2,227	0	2,004	223	2,227
Total	12,400	12,659	13,430	2,095	10,064	1,272	11,335

3.2 WORK ZONE AND DIVERSION ROUTES

Fig. 2 shows travel routes that are important to calculations of extra distance and time that may be required during construction, relative to normal travel when there is no work zone. We differentiate between a method route and a normal route for work zone travel and for diversion travel. Vehicles travel a method route while a construction method is being applied. Diversion method distance = the route vehicles follow between leaving and returning to the normal route if they divert around the work zone. Work zone method distance = zone method distance = distance vehicles follow from one side of the work zone to the other if they do not divert. The normal route is the comparable route vehicles travel when there is no construction. Therefore, diversion normal distance = distance on the normal route between the points at which diverting vehicles leave and return to the normal route. Work zone normal distance = zone normal distance = distance on the normal route between one side of the work zone to the other when there is no work

³ Fig. 1, Fig. 5, and Fig. 8 show the format of CO³ software input.

zone. Fig. 2 (a) shows the distances when there is no detour, in which case work zone method distance = work zone normal distance. Fig. 2 (b) shows the distances where there is a detour and no vehicles travel the normal route through the work zone, in which case generally the work zone method distance > work zone normal distance.

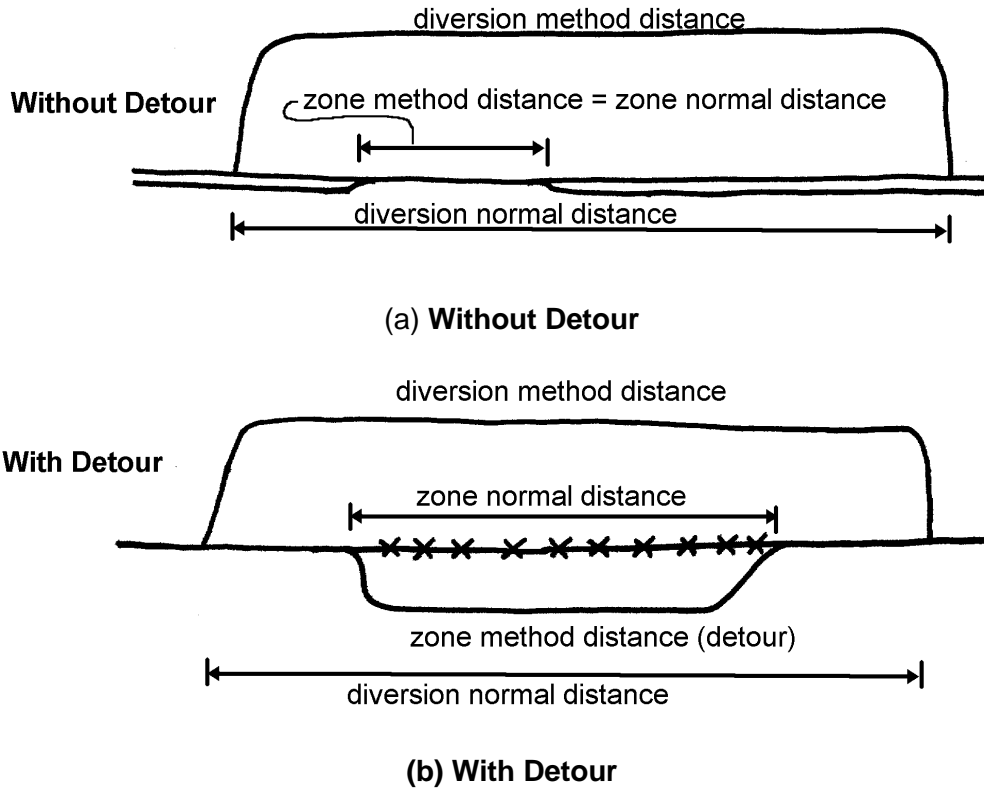


Fig. 2 - Work Zone and Diversion Distances

3.3 DELAY

A work zone can cause three types of delay:

- (1) speed delay = difference in time to travel the method travel distance through the work zone (or around it if a detour is required) during construction and the normal time when there is no work zone..
- (2) backup delay = time vehicles wait to enter the work zone when work and traffic maintenance conditions reduce capacity below design demand, and
- (3) diversion delay = difference in time to travel the diversion travel distance around the work zone during construction and the normal time when there is no work zone.

Vehicles that travel through the work zone experience

work zone delay = speed delay + backup delay (2)

whereas vehicles that divert around the work zone experience diversion delay.

3.4 ACTUAL DEMAND

Demand through the work zone (or detour) decreases when drivers divert to other routes or cancel trips to avoid the work zone conditions and possible delays associated with it. Actual demand in a period consists of vehicles that arrive at the work zone during the period, which is de-

sign demand minus vehicles that cancel trips or divert around the work zone. At any capacity, the decrease in traffic from diverted and canceled vehicles is modeled in four components for both cars and trucks:

- decrease (with no delay) = diverted (with no delay) + canceled (with no delay) traffic that occurs even in periods in which there is no significant delay at the work zone. This is expressed in percent.
- decrease (with delay) = diverted (with delay) + canceled (with delay) traffic that occurs due to delay at the work zone. This is expressed in percent decrease per minute of delay.

Therefore, demand = actual demand is a function of work zone delay:

$$\text{decrease (\%)} = [\text{decrease (with no delay) (\%)}] + [\text{decrease (with delay) (\% / min)}] * [\text{work zone delay (min)}] \dots\dots\dots (3)$$

where **decrease (%)** = per cent of design demand that diverts or cancels its trips. This is demonstrated in Fig. 3 (a) for diverted cars. Total decreases are the sum of diverted and cancelled cars and trucks,

Demand decrease = decrease in design demand is a function of decrease (%) and design demand,

$$\text{decrease (Vph)} = [\text{decrease (\%)}] * [\text{design demand (Vph)}] \\ = \left\{ \begin{array}{l} [\text{decrease (with no delay) (\%)}] \\ + [\text{decrease (with delay) (\% / min)}] * [\text{work zone delay (min)}] \end{array} \right\} * [\text{design demand (Vph)}] \dots\dots\dots (4)$$

and demand = actual demand is calculated by

$$\text{actual demand (Vph)} = [\text{design demand (Vph)}] - [\text{decrease (Vph)}] \\ = [\text{design demand (Vph)}] * \{1 - [\text{decrease (\%)}]\} \dots\dots\dots (5)$$

4 SPEED DELAY

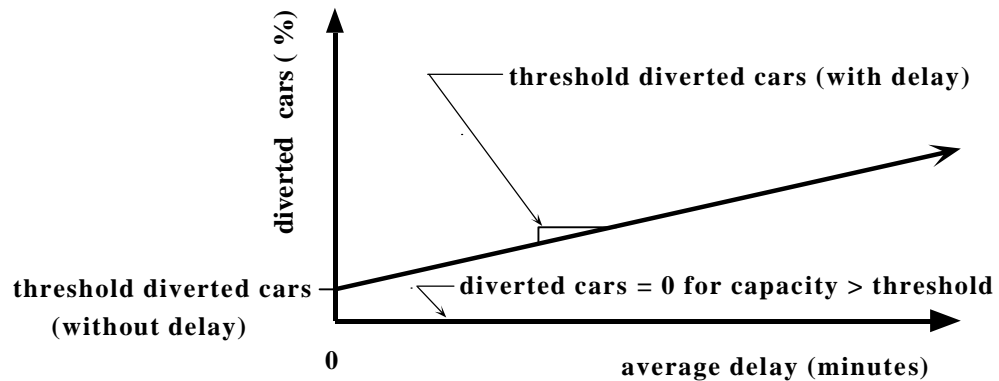
4.1 GENERAL

Vehicle speed through a construction zone may be lower than speeds outside the zone, due to pavement conditions, traffic conditions, routing through a detour, or other work zone conditions. If a detour is used, it adds distance and time. The increase in time required to traverse the work zone is speed delay, and this section describes input variables and calculation of speed delay.

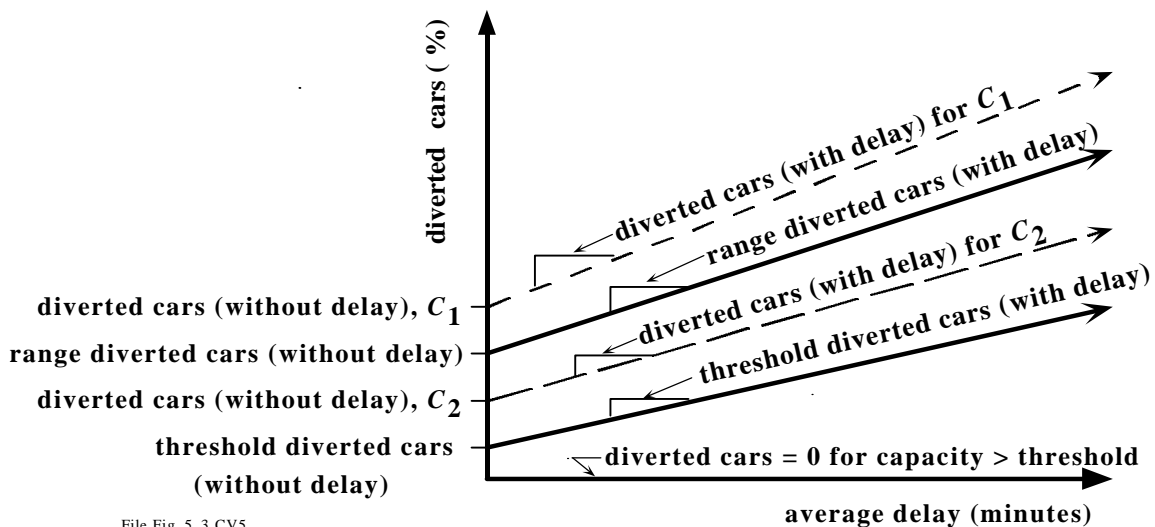
4.2 SPEED DELAY = FUNCTION OF DEMAND AND CAPACITY

Speed delay is the difference in time to travel the method travel distance through the work zone (or around it if a detour is required) during construction and the time it would take if there were no construction. Threshold capacity for speed delay is the capacity at and below which speed delay occurs. Generally, threshold capacity for speed delay for a day is the capacity through the work zone that day during construction. If there are different capacities at different times during a day of construction, the threshold capacity for speed delay is generally the largest of them. Speed delay is calculated for all periods with capacity ≤ (threshold capacity for speed delay), which allows speed delay of a vehicle to vary as work zone conditions and traffic conditions

change. Work zone conditions change from time to time as lanes or ramps are closed and opened or as traffic is routed over temporary paving or detours. Therefore, capacity changes occur when work zone conditions change. Traffic conditions change as work zone conditions change and as demand changes.



(a) With threshold capacity for decreases

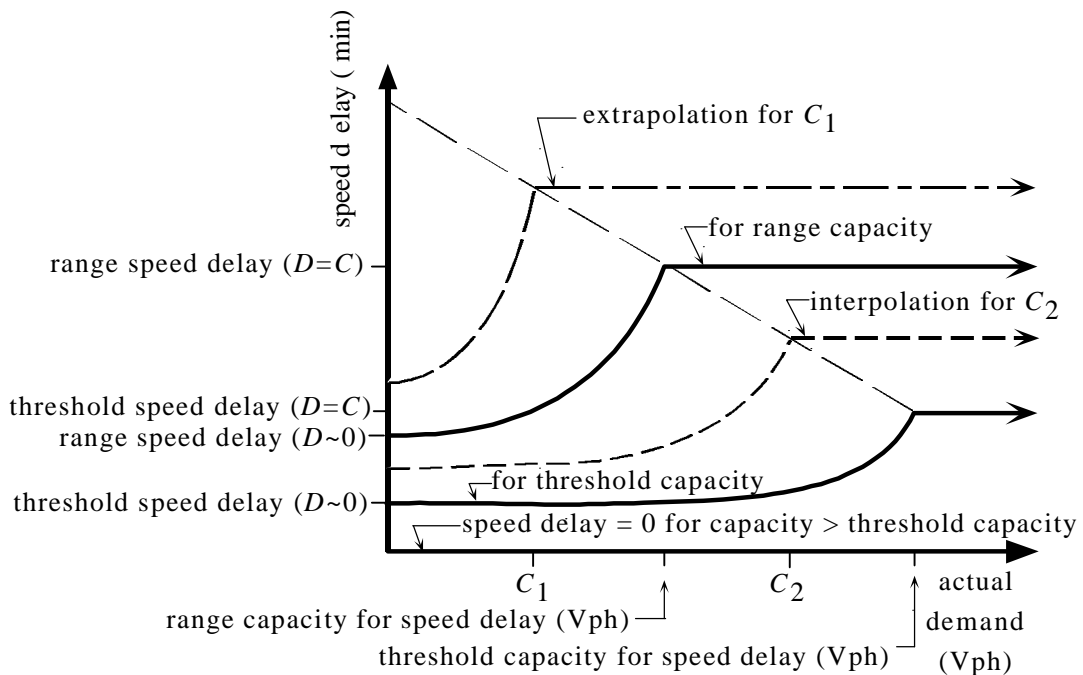
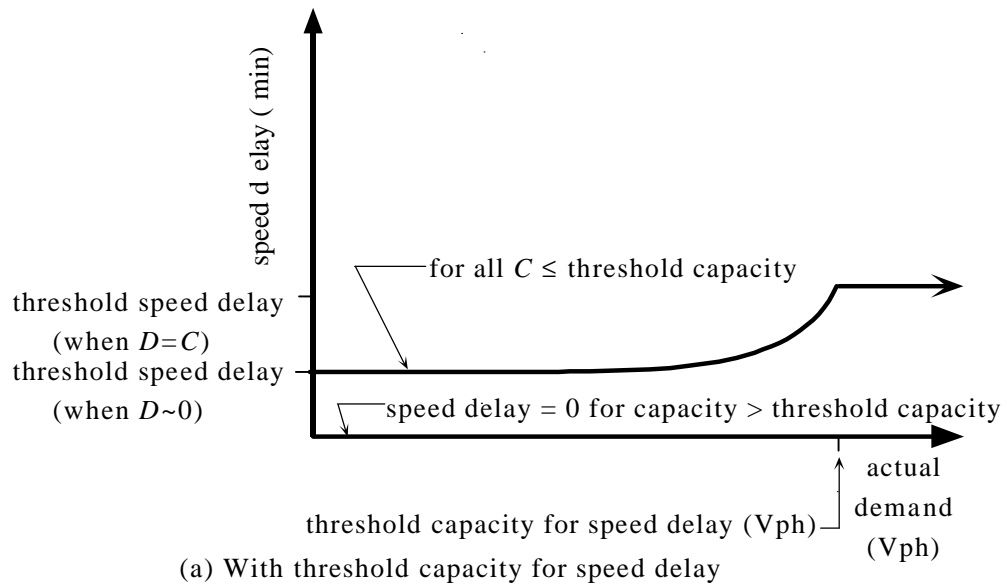


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(b) With threshold and range capacities for decreases

Fig. 3 Percent Decrease in Car Demand for Threshold and Range Values

Speed delay is modeled as a function of period capacity (C) and period zone demand (D), where period zone demand = vehicles that enter the work zone during a period (as opposed to period actual demand = vehicles that arrive at the work zone during a period). That is, each period with capacity \leq (threshold capacity for speed delay) has a speed delay function. For each period the value of its speed delay is a function of the ratio $D/C =$ (demand that enters the zone during the period) / (period capacity), and D/C can have any value between 0 and 1. Fig. 4 shows this relationship between speed delay and zone demand. Speed delay never exceeds its value at $D/C = 1$, because traffic flow through the zone cannot exceed capacity, by definition, and all demand greater than capacity backs up and waits to enter the zone.



File FIG_4B.CV5

Fig. 4 Speed Delay as Function of Capacity and Demand

The speed delay function for each period with capacity \leq (threshold capacity for speed delay) is therefore defined by three variables, which are shown in Speed Delay Input in Fig. 5 (a) for threshold values:

- capacity for speed delay = capacity for the period.
- speed (when $D \sim 0$) = speed when demand is very low.
- speed (when $D = C$) = speed when the number of vehicles that enter the zone during the period equals its capacity.

Fig. 5 (a) shows these values where (threshold capacity for speed delay) = 1400 Vph. For cases where different periods during a day have different capacities and the different capacities

allow different speeds that differ from normal travel speed, another set of values, called **range inputs**, is needed in addition to threshold inputs. Fig. 5 (b) shows values of the three inputs for both threshold values and range values. When there are range inputs, the computer calculates speed delay by interpolating between the threshold inputs for the threshold capacity and the range inputs for the range capacity.

4.3 THRESHOLD CAPACITY FOR SPEED DELAY

4.3.1 Normal Travel

Travel time is a function of travel distance and speed,

$$\text{time} = \frac{\text{distance}}{\text{speed}} \dots\dots\dots (6)$$

For the highway under study, we estimate normal traffic speed = 70 mph when there is no construction work zone. The project considered will require work zone distance = 2 mi, for which normal travel time is

$$\begin{aligned} \text{normal time} &= \frac{\text{work zone normal distance}}{\text{work zone normal speed}} \dots\dots\dots (7) \\ &= \frac{2 \text{ mi}}{70 \text{ mph}} = 0.02857 \text{ hr} = 1.71 \text{ min.} \end{aligned}$$

4.3.2 Low Demand ($D \approx 0$)

We estimate that closing one lane of the example two-lane highway during construction will reduce capacity to 1400 Vph over the two miles of lane closure. At capacity = 1400 Vph, we estimate average vehicle speed at low traffic will be reduced from normal speed = 70 mph to [method speed (when $D \approx 0$)] = 50 mph due to construction conditions.

Travel time during construction at low traffic is calculated from:

$$\begin{aligned} \text{method time (when } D \approx 0) &= \frac{\text{work zone method distance}}{\text{work zone method speed (when } D \approx 0)} \dots\dots\dots (8) \\ &= \frac{2 \text{ mi}}{50 \text{ mph}} = 0.04 \text{ hr} = 2.40 \text{ min.} \end{aligned}$$

and

$$\text{delay} = (\text{method time}) - (\text{normal time}) \dots\dots\dots (9)$$

From this, speed delay at low traffic is calculated below and shown as work zone travel in Fig. 5.

$$\begin{aligned} \text{speed delay (when } D \approx 0) &= [\text{method time (when } D \approx 0)] - [\text{normal time}] \dots\dots\dots (10) \\ &= 2.40 - 1.71 = 0.69 \text{ min} \end{aligned}$$

VEHICLE INPUT	cars	trucks
user cost per hour (\$/V hr)	\$12.00	\$30.00
user cost per mile, (\$/V mi)	\$0.30	\$1.00
user cost per cancellation, (\$/V)	\$4.00	\$10.00
DISTANCE AND SPEED INPUT	distance	speed
work zone method travel	2.0	see delay
work zone normal travel	2.0	70
diversion method travel	10	45
diversion normal travel	4.0	70
SPEED DELAY INPUT	threshold	range
capacity for speed delay (V/hr)	1400	
speed (when D=0) (mph)	50	
speed (when D=C) (mph)	40	
WORK ZONE TRAVEL	threshold	range
method travel time (when D=0) (min)	2.40	
normal travel time (min)	1.71	
speed delay (when D=0) (min)	0.69	
method travel time (when D=C) (min)	3.00	
speed delay (when D=C) (min)	1.29	
WORK ZONE SPEED DELAY USER COST	threshold	range
car speed delay user cost (when D=0)	\$0.14	
truck speed delay user cost (when D=0)	\$0.34	
car speed delay user cost (when D=C)	\$0.26	
truck speed delay user cost (when D=C)	\$0.64	
DIVERSION TRAVEL		
method travel time (min)	13.33	
normal travel time (min)	3.43	
diversion delay (min)	9.90	
extra diversion travel distance (mi)	6.0	
DIVERSION USER COST	cars	trucks
diversion delay user cost	\$1.98	\$4.95
diversion distance user cost	\$1.80	\$6.00
diversion user cost	\$3.78	\$10.95
backup delay balance (min)	17.62	20.62

(a) With threshold capacity

VEHICLE INPUT	cars	trucks
user cost per hour (\$/V hr)	\$12.00	\$30.00
user cost per mile, (\$/V mi)	\$0.30	\$1.00
user cost per cancellation, (\$/V)	\$4.00	\$10.00
DISTANCE AND SPEED INPUT	distance	speed
work zone method travel	2.0	see delay
work zone normal travel	2.0	70
diversion method travel	10	45
diversion normal travel	4.0	70
SPEED DELAY INPUT	threshold	range
capacity for speed delay (V/hr)	1400	1000
speed (when D=0) (mph)	50	45
speed (when D=C) (mph)	40	20
WORK ZONE TRAVEL	threshold	range
method travel time (when D=0) (min)	2.40	2.67
normal travel time (min)	1.71	1.71
speed delay (when D=0) (min)	0.69	0.95
method travel time (when D=C) (min)	3.00	6.00
speed delay (when D=C) (min)	1.29	4.29
WORK ZONE SPEED DELAY USER COST	threshold	range
car speed delay user cost (when D=0)	\$0.14	\$0.19
truck speed delay user cost (when D=0)	\$0.34	\$0.48
car speed delay user cost (when D=C)	\$0.26	\$0.86
truck speed delay user cost (when D=C)	\$0.64	\$2.14
DIVERSION TRAVEL		
method travel time (min)	13.33	
normal travel time (min)	3.43	
diversion delay (min)	9.90	
extra diversion travel distance (mi)	6.0	
DIVERSION USER COST	cars	trucks
diversion delay user cost	\$1.98	\$4.95
diversion distance user cost	\$1.80	\$6.00
diversion user cost	\$3.78	\$10.95
backup delay balance (min)	17.62	20.62

(b) With threshold and range capacity

Fig. 5 - Speed Delay, Diversion, and User Cost

4.3.3 High Demand (D=C)

Travel time during construction at high traffic is calculated below and shown as work zone travel in Fig. 5.

$$\begin{aligned} \text{method time (when } D = C) &= \frac{\text{work zone method distance}}{\text{work zone method speed (when } D = C)} \dots\dots\dots (11) \\ &= \frac{2 \text{ mi}}{40 \text{ mph}} = 0.05 \text{ hr} = 3.00 \text{ min.} \end{aligned}$$

$$\begin{aligned} \text{speed delay (when } D = C) &= [\text{method time (when } D = C)] - [\text{normal time}] \dots\dots\dots (12) \\ &= 3.00 - 1.71 = 1.29 \text{ min} \end{aligned}$$

Col. 4 of Table 3 shows average speed delay = 1.29 min for 9:00 am to 11:00 am, where capacity = 1,400 Vph < actual demand = 2,013 Vph, and for 10:00 am to 12:00 pm, where there is backup for the entire period and traffic flow through the zone = capacity = 1,400 Vph.

4.3.4 Intermediate Demand ($0 < D < C$)

The general equation for speed delay as a function of zone demand = D and capacity = C is
 speed delay = [speed delay (when $D \approx 0$)]

$$+ \left\{ \begin{array}{l} \text{[speed delay (when } D = C)] \\ -\text{[speed delay (when } D \approx 0)] \end{array} \right\} * \left(\frac{D}{C} \right)^{E_s} \quad \text{for } D < C \dots\dots\dots (13)$$

$$= \text{[speed delay (when } D = C)] \quad \text{for } D \geq C$$

where E_s = speed delay exponent (default $E_s = 2$). Fig. 4 illustrates Eq. (13).

Speed delay at an intermediate demand between zero and capacity, such as at $D = 900$ VPH at which $D/C = 900/1400 = 0.643$, is calculated using Eq. (13),

$$\text{speed delay (when } D = 900) = 0.69 + (1.29 - 0.60) * \left(\frac{900}{1400} \right)^2$$

$$= 0.69 + (0.69) * (0.643)^2 = 0.69 + 0.29 = 0.98 \text{ min}$$

4.4 RANGE CAPACITY FOR SPEED DELAY

As described in Sec. 4.2 above, we sometimes need range inputs to describe vehicle speeds at one or more capacities below the threshold capacity for speed delay. For example, outside of working hours, capacity and speeds of a signed work zone are often lower than normal, and capacity and speeds during work periods are even lower. Or capacity and speed may be higher while a lane is being closed than when work is proceeding close to traffic. An example of such range input is shown in Fig. 5 (b). Fig. 4 (b) shows range speed delay and its relationship to threshold speed delay. Range travel time and speed delay follow Eq. (6) to Eq. (12), which produces the range travel times and speed delays shown for work zone travel in Fig. 5 (b). Range speed delay for intermediate demand ($0 < D < C$) follows Eq. (13). For example, this produces [range speed delay (when demand = 900 Vph)] = 3.66 min.

4.5 CAPACITY \neq THRESHOLD CAPACITY OR RANGE CAPACITY

When there is range input as well as threshold input, CO³ calculates values of any variable x_C = value of x at capacity = $C <$ threshold capacity by the following interpolation:

$$x_C = (\text{threshold } x) + [(\text{range } x) - (\text{threshold } x)] * \left[\frac{(\text{threshold capacity}) - C}{(\text{threshold capacity}) - (\text{range capacity})} \right] \dots (14)$$

where (threshold x) = value of x at threshold capacity and (range x) = value of x at range capacity.

The dashed lines in Fig. 4 (b) show interpolated values for capacity C_2 , where (threshold capacity) $> C_2 >$ (range capacity) and extrapolated values for capacity $C_1 <$ (range capacity). Thus for our example we can model any capacities below 1400 Vph. For example, we can calculate travel time and speed delay for capacity = 900 Vph using Eq. (14),

$$\text{method time (when } D \approx 0) = 2.40 + [2.67 - 2.40] * \left[\frac{1400 - 900}{1400 - 1000} \right]$$

$$= 2.40 + [0.27] * [1.25] = 2.40 + 0.33 = 2.73 \text{ min}$$

$$\begin{aligned} \text{method time (when } D = C) &= 3.00 + [(6.00) - (3.00)] * \left[\frac{1400 - 900}{1400 - 1000} \right] \\ &= 3.00 + [3.00] * [1.25] = 3.00 + 3.75 = 6.75 \text{ min} \end{aligned}$$

These are 2.0 mi travel times at [average speed (when $D \approx 0$)] = (2.0 mi) / [(2.73 min)*(60 min/hr)] = 38.1 mph and [average speed (when $D=C$)] = 17.8 mph. Similarly, Eq. (14) calculates speed delay for capacity = 900 Vph,

$$\text{speed delay (when } D \approx 0) = 0.69 + [0.95 - 0.69] * \left[\frac{1400 - 900}{1400 - 1000} \right] = 1.02 \text{ min}$$

$$\text{speed delay (when } D = C) = 1.29 + [4.29 - 1.29] * \left[\frac{1400 - 900}{1400 - 1000} \right] = 5.04 \text{ min}$$

which agree with Eq. (12), from which [speed delay (when $D \approx 0$)] = 2.73 - 1.71 = 1.02 min, and [speed delay (when $D = C$)] = 6.75 - 1.71 = 5.04 min.

4.6 USER COST OF SPEED DELAY

User cost = monetary equivalent of any extra travel time and distance that vehicles incur due to construction. We calculate speed delay user cost using Eq. (15), for which Fig. 5 shows an example of user cost per hour (\$/V-hr) and user cost per mile (\$/V-mi).

$$\begin{aligned} \text{user cost} &= [\text{delay (hr)}] * [\text{user cost per hour (\$/hr)}] \\ &+ \{ [\text{method distance (mi)}] - [\text{normal distance (mi)}] \} * [\text{user cost per mile (\$/mi)}] \dots (15) \end{aligned}$$

For example,

$$\begin{aligned} \text{car speed delay user cost (when } D = C) &= \frac{1.29 \text{ min}}{60 \text{ min/hr}} * (\$12.00 / \text{hr}) \\ &+ (2.0 \text{ mi} - 2.0 \text{ mi}) * (\$0.30 / \text{mi}) = \$0.26 \end{aligned}$$

Similarly, truck speed delay user cost (when $D = C$) = (1.29 min) * (\$30.00 / hr) / (60 min/hr) = \$0.645. Speed delay user cost (\$/V) for other conditions for cars and trucks is shown in Fig. 5.

5 BACKUP DELAY

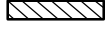
5.1 GENERAL


A backup is a queue of vehicles that is waiting to enter a work zone, and backup delay is the time a vehicle waits in the queue to enter the zone. CO³ assumes arrivals and departures are uniformly distributed over each period. Backup is the excess of demand over capacity, and all vehicles ahead of a particular vehicle must pass through the zone before it can. Therefore, a vehicle that arrives when there is a backup must wait to enter the zone until the backup ahead of it has entered the zone. If there are B_N vehicles backed-up when vehicle N arrives, and the capacity (V/period) of the zone is capacity_N following the arrival of N , vehicle N is delayed by (backup delay) _{B_N} while it waits to enter the zone, where

$$[\text{backup delay (min)}]_{B_N} = \frac{[B_N(V)] [60 \text{ (min/hr)}]}{[\text{capacity}_N(\text{Vph})]} \dots (16)$$

These relationships are shown in Fig. 6, in which $B_{sop\ i}$ = backup at start-of-period i , and $B_{eop\ i}$ = backup at end-of-period i . Of course, the start-of-period backup of one period = end-of-period backup of the previous period, or

$$B_{sop\ i+1} = B_{eop\ i} \dots\dots\dots (17)$$

Vehicles arriving at the start-of-period 1 are behind backup $B_{sop\ 1}$. The last vehicle in that backup enters the zone at time **a**, after which time the first vehicles arriving in period 1 will start entering the zone. The number of period 1 vehicles that enter the zone during period 1 = $e - d$. If period 1 demand = demand_{1,1}, then end-of-period 1 backup = $B_{eop\ 1,1}$ = start-of-period 2 backup = $B_{sop\ 2,1}$, and the last arriving period 1 vehicle will be backed-up until time **b**, at which time all of demand_{1,1} will have entered the zone. Thus, all period 1 vehicles will have entered the zone before end-of-period 2, and end-of-period 2 backup = $B_{eop\ 1,1} = 0$. The total vehicle-periods period 1 vehicles will have spent in backups is the cross-hatched area  = (total backup delay)_{1,1}.

However, if period 1 demand = demand_{1,2}, then period 1 vehicles will remain into period 3, the latest arriving period 1 vehicles will be backed-up from end-of-period 1 until **c**, and the total vehicle-periods of backup delay accumulated by period 1 vehicles will be the area  = (total backup delay)_{1,2}.

Thus, the total vehicle-periods of delay due to backups experienced by vehicles that arrive in a period is a function of the backup at the start of the period, the period demand, and the period capacity. It is also a function of the following periods' capacities, if the initial start-of-period backup and period demand exceed the period capacity.

5.2 BACKUP AS FUNCTION OF DEMAND AND CAPACITY

The relationship among these variables, as demand and capacity vary from period to period, is shown in Fig. 7. The slopes of the dashed line along the top of the crosshatched areas represent the period demands, and the slopes of the solid line along the bottom represent the period capacities. Each point along the demand line represents arrival of a vehicle at the zone on the vertical axis and the time the vehicle arrives on the horizontal axis. A horizontal line drawn from that point intersects the capacity line at the time the vehicle enters the zone. The horizontal distance between the point of arrival on the demand line and the point of entering the zone on the capacity line represents the backup delay for the vehicle. For example, the vehicle that arrives at 9 (point A) will enter the zone as shown by point B, and it will be backed up for time AB.

A period's demand does not start to enter the work zone (or other primary path) until all preceding backed-up vehicles have entered the zone. Thus, the first vehicles arriving 7-8 can enter the zone immediately. However, no vehicles arriving 9-10 can enter the zone until well into time 11-12.

Similarly, the last vehicle to arrive in a period is backed-up until the earlier arriving vehicles, including those backed-up from prior periods, have entered the zone. Thus, vehicles that arrive just before 8 will be backed-up until almost 9, because demand₇₋₈ significantly exceeds capacity₇₋₉. Vehicles arriving just before 10 will be backed-up until past 12, because the backup at 9 plus demand₉₋₁₀ exceeds capacity₉₋₁₂.

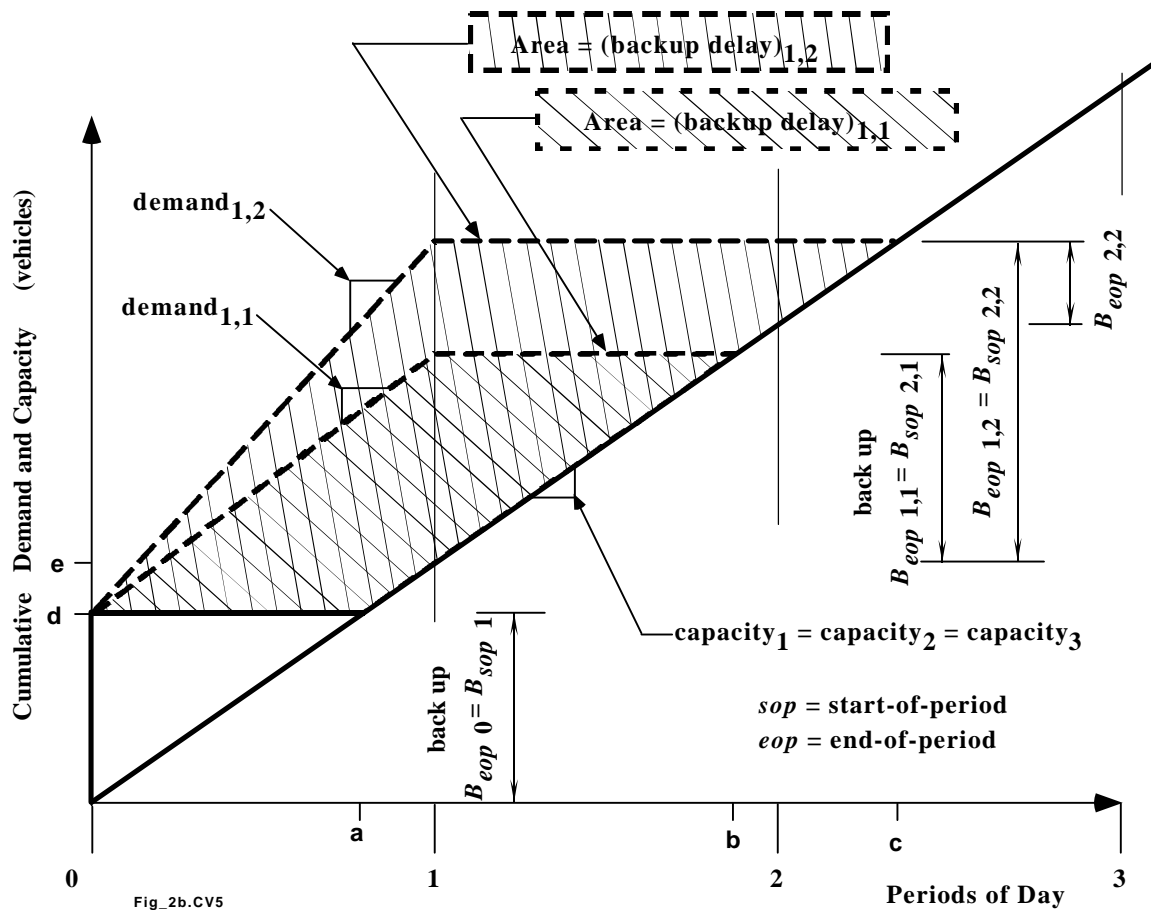


Fig. 6 Comparison of Backup Delay for Two Demands

The CO^3 model determines the vehicle-periods of delay for the each period's demand by tracking the period demand through successive periods until all of its vehicles have entered the zone. CO^3 first calculates the period's vehicles' delays for the period in which they arrive. If vehicles remain at the end of the period, those vehicles' delays are calculated for the succeeding period. If vehicles remain at the end of the succeeding period, their delays are calculated for the next succeeding period. This continues until none of the initial period's vehicles remain. The sum of the delays for those periods is the total backup delay of the vehicles that arrived in the initial period. If the periods are one hour in length, then total backup delay is measured in vehicle-hours (V-hr).

5.3 INITIAL PERIOD AND FOLLOWING PERIOD CONDITIONS

The initial and following period backup conditions for vehicles arriving during a period are indicated by the following terms:

- I = Initial, indicating the initial period = the period in which the demand occurs.
- F = Following, indicating a period following the period in which the demand occurs.
- B indicates a start-of-period backup $<$ (D = initial period demand).
- D indicates a following period start-of-period backup \geq (D = initial period demand).
- R indicates $0 <$ end-of-period backup $<$ (D = initial period demand).
- O indicates there is no backup, i.e., backup = 0.

These terms are combined to describe the different possible conditions for which to calculate period backup and delay. The start-of-period and end-of-period condition classes are described in Table 2.

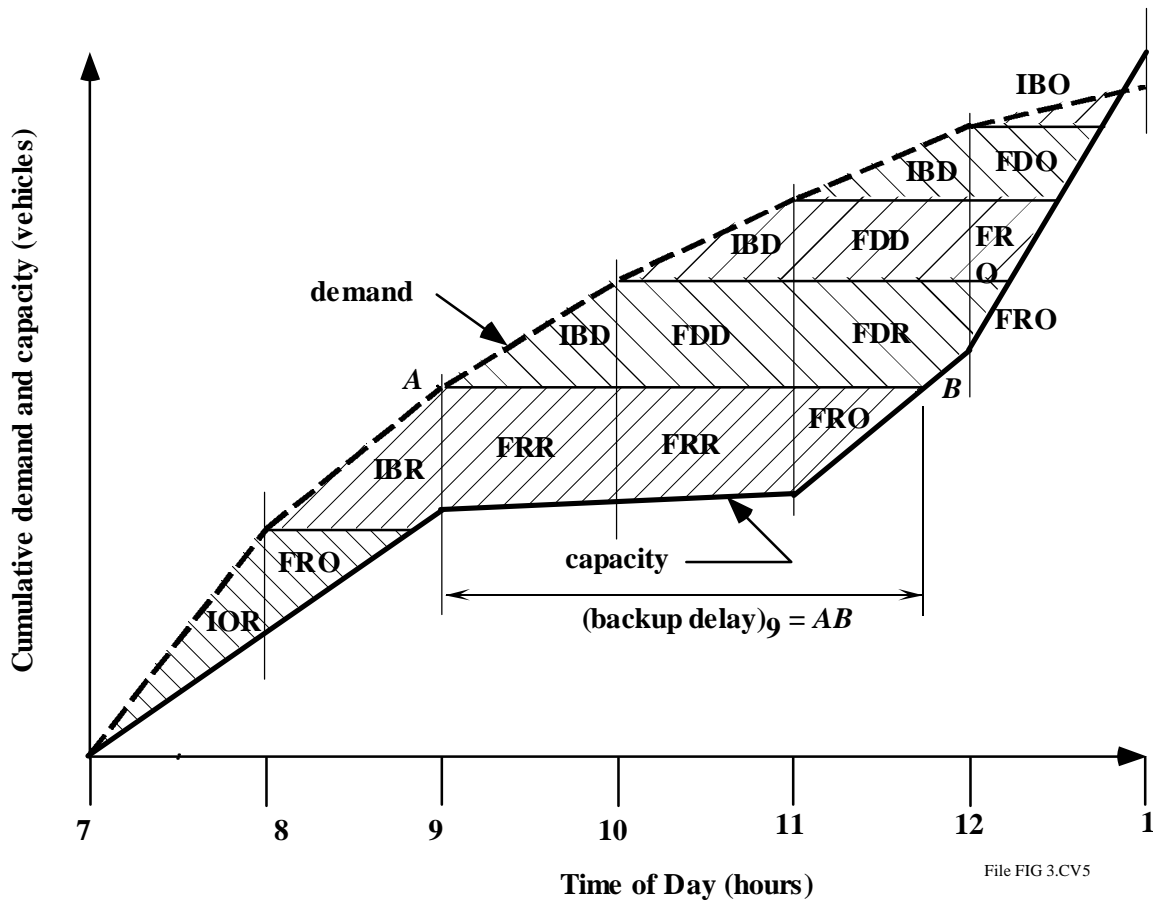


Fig. 7. Backup, from Cumulative Demand and Capacity

Period 1, which is the initial period considered in Fig. 6, is IBR for both demand_{1,1} and demand_{1,2}, because it has start-of-period backup = $d > 0$ and end-of-period backup > 0 . Period 2, which follows period 1, is FRO for demand_{1,1}, because period 2 capacity exceeds period 1 end-of-period backup, and no period 1 demand is backed up at end-of-period 2. However, if period 1 demand = demand_{1,2}, then period 2 is FRR and period 3 is FRO, because the last part of period 1 demand is still backed up at end-of-period 2 but all of it has entered at end-of-period 3.

Each period in Fig. 7 is labeled to show its condition. Fig. 7 shows each of the nine possible period conditions, with several possible condition sequences. For example, period 9-10 demand proceeds through sequence IBD, FDD, FDR, and FRO in periods 9-10, 10-11, 11-12, 12-1 respectively.

For each period i , the sum of initial period demand and start-of-period backup is compared to initial period capacity to determine the initial period class. The initial period delay and end-of-period backup are calculated using formulas appropriate to that class. If the initial period end-of-period backup is greater than 0 (i.e., if $B_{eop\ I} = B_{eop\ i} > 0$), then end-of-period backup is compared to the following period capacity to determine the following period class. The following period delay and end-of-period backup are calculated using formulas appropriate to that class. This continues from following period to following period until all the initial period demand has been accommodated (i.e., until $B_{eop\ F} = 0$). At this point $(\text{backup delay})_i = \text{total backup delay for ve-}$

icles that arrive during period i and $(\text{average backup delay})_i =$ average backup delay per vehicle that arrives during period i can be calculated.

$$(\text{backup delay})_i = \sum_{\substack{i,j \\ j=i,i+1,i+2\dots}} (\text{backup delay})_{i,j} \dots\dots\dots (18)$$

$$(\text{average backup delay})_i = \frac{(\text{backup delay})_i}{(\text{actual demand})_i} = \frac{(\text{backup delay})_i}{D_i} \dots\dots\dots (19)$$

where $(\text{backup delay})_{i,j} =$ the delay during period j , for vehicles that arrived in period i .

Table 2. Period Backup Condition Relationships

Class (1)	Start of Period		End of Period	
	Symbol (2)	Condition (3)	Symbol (4)	Condition (5)
(a) Start of period and end of period backup for I = initial period				
IOR	O	$B_{sop \cdot I} = 0$	R	$D_I > B_{eop \cdot I} > 0$
IOO			O	$B_{eop \cdot I} = 0$
IBD	B	$B_{sop \cdot I} > 0$	D	$B_{eop \cdot I} \geq D_I$
IBR			R	$D_I > B_{eop \cdot I} > 0$
IBO			O	$B_{eop \cdot I} = 0$
(b) Start of period and end of period backup for F = following period				
FDD	D	$B_{sop \cdot F} \geq D_I$	D	$B_{eop \cdot F} \geq D_I$
FDR			R	$D_I > B_{eop \cdot F} > 0$
FDO			O	$B_{eop \cdot F} = 0$
FRR	R	$D_I > B_{sop \cdot F} \geq 0$	R	$D_I > B_{eop \cdot F} > 0$
FRO			O	$B_{eop \cdot F} = 0$

5.4 PERIOD BACKUP FORMULAS

For each initial period, $B_{eop \cdot I} =$ (end-of-period backup)_I is calculated from

$$B_{eop \cdot I} = B_{sop \cdot I} + D - C \quad \text{for } B_{sop \cdot I} + D > C \dots\dots\dots (20)$$

$$B_{eop \cdot I} = 0 \quad \text{for } B_{sop \cdot I} + D \leq C \dots\dots\dots (21)$$

Classes IOR, IBD, and IBR follow Eq. (20), and class IBO follows Eq. (21) and has no end-of-period backup. Similarly, we calculate $B_{eop \cdot F}$ of each following period, using

$$B_{eop \cdot F} = B_{sop \cdot F} - C \quad \text{for } B_{sop \cdot F} > C \dots\dots\dots (22)$$

$$B_{eop \cdot F} = 0 \quad \text{for } B_{sop \cdot F} \leq C \dots\dots\dots (23)$$

Classes FDD, FDR, and FRR follow Eq. (22), and classes FDO and FRO follow Eq. (23) and have no end-of-period backup. Appendix 2 shows the period demand, capacity, start-of-period

backup, and end-of-period backup for each initial period and following backup condition described in Table 2.

5.5 BACKUP DELAY EXAMPLE

Table 1 shows an example of capacity and historical demand inputs into CO³ and calculated values for design demand from 8:00 am (period 8A) through 2:00 pm (period 1P) using Eq. (1). Normal capacity is 3,400 Vph, and work zone capacity is reduced to 1,400 Vph due to lane closure from 9A to 12P. Table 1 also shows calculated actual demand, and Table 3 shows backup and delay times. CO³ calculation of backup demand is shown in this section, and calculation of actual demand is described in Sec. 6.

CO³ calculates each period's backup delay from its capacity and actual demand and from capacity in the following period(s) if vehicles are backed up at end-of-period. Drawings and equations in Appendix 2 shows relationships of actual demand, capacity, backup, backup delay, and speed delay for the period classes described in Table 2.

Though Table 1 does not show values before 8A, (capacity before 9A) = 3,400 Vph, which exceeds both threshold capacity for speed delay and design demand. Therefore, before 9A there is no speed delay or backup delay, actual demand = design demand, and all periods before 9A are class IOO.

In Col. 2 of Table 3 we see period 9A has no start-of-period backup, because 8A end-of-period backup = 0. We use Eq. (20) to calculate 9A end-of-period backup = $B_{eop} 9A = 0 + 2,013 - 1,400 = 613$ V, as shown in Col. 2 in Table 3 for 9A. Period 9A is class IOB, because it is an initial period, $B_{sop} 9A = 0$, and demand = $D_{9A} = 2,013 > B_{eop} 9A = 613 > 0$. From Eq. (38),

$$(\text{backup delay})_{9A,9A} = \frac{D - C}{2} = \frac{2,013 - 1,400}{2} = 307 \text{ V - hr}$$

By Eq. (23), For 9A demand, period 10A is class FRO, because it follows 9A, it has capacity = $C = 1,400 > B_{sop} 10A = B_{eop} 9A = 613$, and by Eq. (23) $B_{eop} 10A = 0$ because $B_{sop} 10A < \text{capacity}_{10A} = 1,400$. Therefore, from Eq. (53),

$$(\text{backup delay})_{9A,10A} = \frac{B_{sop}^2}{2 * C} = \frac{(613)^2}{2 * 1,400} = 134 \text{ V - hr}$$

From Eq. (18) and (19),

$$(\text{backup delay})_{9A} = (\text{backup delay})_{9A,9A} + (\text{backup delay})_{9A,10A} = 307 + 134 = 441 \text{ V - hr}$$

$$(\text{average backup delay})_{9A} = \frac{(\text{backup delay})_{9A}}{D_{9A}} = \frac{(441 \text{ V - hr}) * (60 \text{ min / hr})}{2013 \text{ V / hr}} = 13.14 \text{ min}$$

as shown in Col. 3 of Table 3.

Similarly, we use Eq. (20) to calculate $B_{eop} 10A = 613 + 1,366 - 1,400 = 579$ Vph, as shown for 10A in Table 3. Therefore, for 10A demand, period 10A is class IBR, because it is an initial period, $B_{sop} 10A = B_{eop} 9A = 613 > 0$, and demand = $D_{10A} = 1,366 > B_{eop} 10A = 579 > 0$. From Eq. (43), $(\text{backup delay})_{10A,10A} = 0.5 * [1,366 - (1,400 - 613)]^2 / 1,400 = 462$ V. For 10A demand, period 11A is class FRO, because it follows 10A, it has capacity = $C_{10A} = 1,400 > B_{sop} 11A = B_{eop} 10A = 579$, and by Eq. (23) $B_{eop} 10A = 0$ because $B_{sop} 11A = 579 < \text{capacity}_{11A} = 1,400$. Therefore, from Eq. (53), $(\text{backup delay})_{10A,11A} = (579)^2 / (2 * 1,400) = 120$

V-hr. From Eq. (18) and (19), (backup delay)_{10A} = 462 + 120 = 582 V-hr, and (average backup delay) _{10A} = (582 V-hr) * (60 min/hr) / (1,366 Vph) = 25.55 min. as shown in Col. 3 of Table 3.

5.6 USER COST OF BACKUP DELAY

In parallel with speed delay user cost, we calculate backup delay user cost using Eq. (15). Method distance and normal distance have no meaning for backup delay. Therefore, backup delay user cost = (backup delay) * (user cost per hour). From this, (car backup user cost)_{9A} = (backup delay)_{9A} * (car user cost per hour) = (13.14 min) * (\$12.00 / hr) / (60 min/hr) = \$2.63. From Col. 6 of Table 1 for (car actual demand)_{9A} = 1,773 Vph, (total car backup user cost)_{9A} = (1,733) * 2.63 = \$4,558. Similarly (truck backup user cost)_{9A} = (backup delay)_{9A} * (truck user cost per hour) = (13.14 min) * (\$30.00 / hr) / (60 min/hr) = \$6.57. From Col. 7 of Table 1 for (truck actual demand)_{9A} = 240 Vph, (total truck backup user cost)_{9A} = (240) * 6.57 = \$1,577. Total backup user cost is the sum of car and truck backup user cost = 4,558 + 1,577 = \$6,135.

Table 3. Period Backup, Delay, and User Cost

period (hr) (1)	end of period backup (V) (2)	average backup delay (min) (3)	average speed delay (min) (4)	average delay (min) (5)	total period delay (V hr) (6)	delay cost ($\$$) (7)	decrease cost ($\$$) (8)	user cost ($\$$) (9)
8 A	0	0.00	0.00	0.00	0	\$0	\$0	\$0
9 A	613	13.14	1.29	14.43	484	\$6,846	\$2,325	\$9,172
10 A	579	25.55	1.29	26.83	611	\$8,846	\$3,316	\$12,162
11 A	271	18.22	1.29	19.50	355	\$5,065	\$1,760	\$6,825
12 P	194	9.63	1.10	10.73	237	\$3,329	\$1,146	\$4,475
1 P	0	0.28	0.00	0.28	11	\$145	\$0	\$145
Total					1,697	\$24,231	\$8,547	\$32,778

5.7 AVERAGE AND TOTAL DELAY AND USER COST

In accord with Eq. (2), average delay for vehicles that arrive at the work zone during a period = average work zone delay = sum of speed delay and backup delay. Period delay = total period delay is calculated,

$$(\text{period delay})_i = (\text{average delay})_i * (\text{actual demand})_i \dots\dots\dots (24)$$

and period delay cost is calculated from Eq. (15). From Table 3, (average delay)_{9A} = 13.14 + 1.29 = 14.43 min = 0.2404 hr. Then (period delay)_{9A} = (0.2404 hr) * 2,013 = 484 hr. Using car and truck actual demand from Table 1, (delay cost)_{9A} = (0.2404 hr) * [(1,773 cars) * (\$12.00 / car-hr) + (240 trucks) * (\$30.00 / truck-hr)] = (0.2404) * [21,276 + 7,200] = (0.2404) * [28,476] = \$6,846, as shown in Col. 7 of Table 3.

6 DECREASES TO DESIGN DEMAND

6.1 DEMAND DECREASE = FUNCTION OF DELAY

CO³ provides for cars and trucks to divert to an alternate route or cancel their trips as vehicles are delayed by backups and speed reductions at work zones. Eq. (3) describes the general equa-

tion for decrease (%) = % decrease in demand, which is demonstrated in Fig. 3 (a) for diverted cars. Total decrease is the sum of diverted and cancelled cars and trucks,

$$\left[\begin{array}{l} \text{decrease (\%)} \\ \text{(with no delay)} \end{array} \right] = \left\{ \begin{array}{l} \left[\text{car decrease (\%)} \text{ (with no delay)} \right] \\ + \left[\text{truck decrease (\%)} \text{ (with no delay)} \right] \end{array} \right\} \\ = \left\{ \begin{array}{l} (\% \text{ cars}) * \left\{ \left[\left[\text{diverted cars (with no delay)} \right] + \left[\text{cancelled cars (with no delay)} \right] \right] \right\} (\%) \\ + (\% \text{ trucks}) * \left\{ \left[\left[\text{diverted trucks (with no delay)} \right] + \left[\text{cancelled trucks (with no delay)} \right] \right] \right\} (\%) \end{array} \right\} \dots\dots\dots (25)$$

$$\left[\begin{array}{l} \text{decrease (\% / min)} \\ \text{(with delay)} \end{array} \right] = \left\{ \begin{array}{l} \left[\text{car decrease (with delay)} \text{ (\% / min)} \right] \\ + \left[\text{truck decrease (with delay)} \text{ (\% / min)} \right] \end{array} \right\} \\ = \left\{ \begin{array}{l} (\% \text{ cars}) * \left\{ \left[\left[\text{diverted cars (with delay)} \right] + \left[\text{cancelled cars (with delay)} \right] \right] \right\} \left(\frac{\%}{\text{min}} \right) \\ + (\% \text{ trucks}) * \left\{ \left[\left[\text{diverted trucks (with delay)} \right] + \left[\text{cancelled trucks (with delay)} \right] \right] \right\} \left(\frac{\%}{\text{min}} \right) \end{array} \right\} \dots\dots\dots (26)$$

Magnitudes of the decrease (%) variables are functions of users' knowledge of conditions (including diversion routes), users' access to diversion routes, and delays encountered by users. Decrease (Vph) = decrease in design demand is a function of decrease (%) and design demand, as shown in Eq. (4), from which demand = actual demand is calculated by Eq. (5).

6.2 THRESHOLD CAPACITY FOR DEMAND DECREASE

If (1) capacity is the same for all periods during construction or if (2) all capacities during construction have the same demand decrease value, then we input only threshold values for demand decreases, as shown in Fig. 8 (a). This shows threshold capacity for decreases to design demand = 1,400 Vph. Fig. 3 (a) shows the relationship between diverted cars and delay for the threshold capacity. To demonstrate the CO³ model, we first start with (car design demand)_{9A} = (car design demand % = 90%) * [(design demand)_{9A} = 2,584] = 2,326 Vph. For capacity_{9A} = threshold capacity for demand decrease = 1,400 Vph, we input diverted car values from Fig. 8 (a) and the (average delay)_{9A} = 14.43 min into Eq. (3),

$$\left[\text{diverted cars (\%)} \right]_{9A} = \left[\text{diverted cars (with no delay)} = 3\% \right] \\ + \left[\text{diverted cars (with delay)} = 1\% / \text{min} \right] * \left[(\text{delay})_{9A} = 14.43 \text{ min} \right] \\ = 3\% + 14.43\% = 17.43\%$$

Therefore, (diverted cars)_{9A} = (17.43%) * (2,326) = 406 V. Similarly, (cancelled cars)_{9A} = [2.0% + (0.3%)(14.43 min) = 2.0 + 4.33 = 6.33%] * (2,326) = 147 V. From this, (car de-

crease) $g_A = 406 + 147 = 553$ Vph, and (actual car demand) $g_A = 2,326 - 553 = 1,773$ Vph, as shown in Col 6, Table 1.

Similarly, using values from Fig. 8 (a) for total traffic, from Eq. (25) [decrease (%) (with no delay)] = (90 %) * (3 % + 2 %) + (10%) * (0 + 0) = 4.5 % and from Eq. (26) [decrease (%/min) (with delay)] = (90 %) * (1 %/min + 0.3 %/min) + (10 %) * (0.5 %/min + 0) = 1.22 %/min. From Eq. (3), total traffic [decrease (%)] $g_A = 4.5 % + (1.22 %/min) * (14.43 \text{ min}) = 4.5 + 17.6 = 22.1 %$. From Eq. (4), traffic (decrease) $g_A = 22.1 % * 2,584 \text{ V} = 571 \text{ V}$, and (actual demand) $g_A = 2,584 - 571 = 2,013 \text{ V}$ as shown in Col. 8 of Table 1.

6.3 RANGE CAPACITY FOR DEMAND DECREASE

In parallel with Sec. 4.4 above, we sometimes need range inputs to describe vehicle decreases at one or more capacities below the threshold capacity for demand decrease. An example of such range input is shown in Fig. 8 (b). Fig. 3 (b) shows range diverted cars and its relationship to threshold diverted cars. Range decreases and actual demand follow Eq. (3) and (5) in parallel with threshold decrease and actual demand calculations.

Also in parallel with speed delay, when there is range input for demand decrease, as well as threshold input, CO³ calculates values of any demand decrease value at capacity = $C < \text{threshold capacity}$ using Eq. (14). The dashed lines in Fig. 3 (b) show interpolated values for capacity C_2 , where (threshold capacity) $> C_2 > (\text{range capacity})$ and extrapolated values for capacity $C_1 < (\text{range capacity})$. Thus for our example we can model any capacities below 1400 Vph. For example, using Eq. (14) we can calculate [diverted cars (with no delay) for capacity = 900 Vph] = 3% + (6% - 3%) * [(1,400 - 900) / (1,400 - 1,000)] = 3% + (3%) * (1.25) = 6.75%.

DECREASE TO DEMAND INPUT	threshold	range
capacity for decreases to design demand (V/hr)	1400	
canceled cars (with no delay) (%)	2.0%	
canceled trucks (with no delay) (%)		
canceled cars (with delay) (%/min)	0.3%	
canceled trucks (with delay) (%/min)		
diverted cars (with no delay) (%)	3.0%	
diverted trucks (with no delay) (%)		
diverted cars (with delay) (%/min)	1.0%	
diverted trucks (with delay) (%/min)	0.5%	

(a) With threshold capacity

DECREASE TO DEMAND INPUT	threshold	range
capacity for decreases to design demand (V/hr)	1400	1000
canceled cars (with no delay) (%)	2.0%	4.0%
canceled trucks (with no delay) (%)		
canceled cars (with delay) (%/min)	0.3%	0.3%
canceled trucks (with delay) (%/min)		
diverted cars (with no delay) (%)	3.0%	6.0%
diverted trucks (with no delay) (%)		
diverted cars (with delay) (%/min)	1.0%	1.0%
diverted trucks (with delay) (%/min)	0.5%	0.5%

(b) With threshold and range capacity

Fig. 8 Input for Decrease to Demand

7 DELAY, DEMAND DECREASE, AND ACTUAL DEMAND

7.1 GENERAL

Speed delay and backup delay are functions of actual demand, which is a function of traffic decrease, and traffic decrease is a function of speed delay and backup delay. This circular dependency requires an iterative solution for work zone delay and actual demand for many scenarios.

CO³ iterative solutions calculate and test successive trial values of actual demand and work zone delay until they converge to an acceptable solution.

However, speed delay occurs only when capacity \leq (threshold capacity for speed delay). Traffic decreases = diversions + cancellations occur only when capacity \leq (threshold capacity for demand decrease). Otherwise, (actual demand) = (design demand). Backup delay occurs only when (actual demand) $>$ capacity. Therefore, we can have speed delay without backup delay or traffic decreases. We can have backup and backup delay without speed delay or traffic decreases. We can have traffic decreases due to [decreases (with no delay)] when there is no speed delay or backup delay. Because of constraints and relationships such as these, we can directly calculate work zone delay, decreases, and actual design for several scenarios without an iterative solution.

For example, Table 1 shows capacity_{8A} = 3,400 Vph, which is larger than (1) (design demand)_{8A}, (2) (threshold capacity for speed delay), and (3) (threshold capacity for demand decrease). Considering that (start-of-period backup)_{8A} = 0, then from the above we know that in period 8A there will be no backup, no speed delay, and no demand decrease.

7.2 COMMON CASE

The most common case of backup, speed delay, and demand decrease is (1) where a period's [(demand) + (start-of-period backup)] $>$ capacity, and (2) where capacity is constant during the time all the period's vehicles enter the zone. We call periods that exhibit these two conditions the Common Case. The hatched area in Fig. 9 and both hatched areas in Fig. 6 satisfy the Common Case. These conditions are also met for demands of periods 7, 10, and 11 in Fig. 7. Periods 8 and 9 are not Common Case, because each demands enters the work zone when there is more than one capacity. For example, demand₈ enters the work zone with capacities from 8 to 12, and capacity₈ \neq capacity₉ = capacity₁₀ \neq capacity₁₁. Period 12 is not Common Case, because demand₁₂ + (start-of-period backup)₁₂ $<$ capacity₁₂.

Similarly, in our example, Table 1 shows demand_{9A} = 2,013 Vph and capacity_{9A} = 1,400 Vph. From this we determine that demand_{9A} + (start-of-period backup)_{9A} = 2,013 + 0 = 2,013 $>$ capacity_{9A} = 1,400 Vph. This satisfies Common Case condition 1. We calculate (end-of-period backup)_{9A} = 2,013 - 1,400 = 613 V, as is also shown in Col. 2 of Table 3. We also see capacity_{10A} = 1,400 Vph $>$ (end-of-period backup)_{9A} = 613, so the 613 V of demand_{9A} remaining at 10A enters the work zone in period 10A. Therefore, all of demand_{9A} enters the work zone under the same capacity_{9A,10A} = 1,400 Vph, which satisfies Common Case condition 2. Therefore, period 9A is Common Case, as are 10A and 11A.

However, 12P is not Common Case. From Eq. (20) we calculate $B_{eop12P} = 271 + 1,323 - 1,400 = 194$ V. Thus the first $1,323 - 194 = 1,129$ V of demand_{12P} enters the zone at capacity_{12P} = 1,400 Vph and the last 194 V of demand_{12P} enters the zone at capacity_{1P} = 3,400 Vph \neq 1,400 Vph. Therefore, Common Case condition 2 is not satisfied, and period 12P is not Common Case.

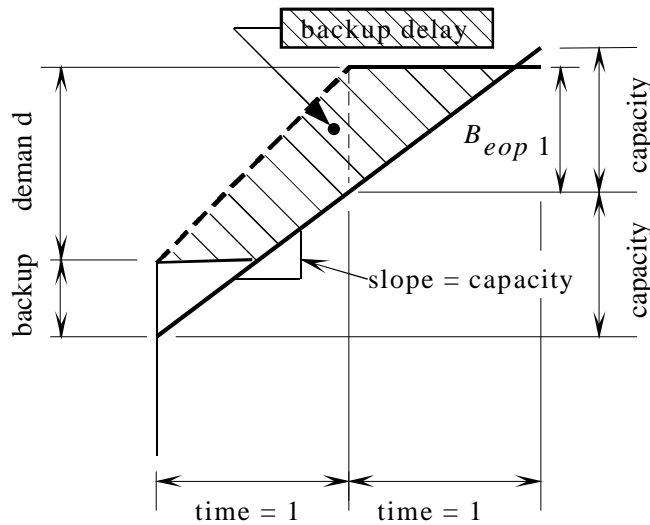


Fig. 9 Constant Capacity for Period's Demand

Each of the hatched areas of Fig. 6, Fig. 7, and Fig. 9 shows total backup delay. For the Common Case periods identified above, the hatched area is a rhomboid, whose area is

$$\text{total backup delay (hr)} = \frac{(\text{demand}) * (\text{demand} + 2 * \text{backup} - \text{capacity})}{2 * (\text{capacity})} \dots \dots \dots (27)$$

from which we can calculate average backup delay per vehicle,

$$\begin{aligned} \text{backup delay (min / V)} &= \frac{(\text{total backup delay}) * (60 \text{ min / hr})}{\text{demand}} \dots \dots \dots (28) \\ &= \frac{30 * (\text{demand} + 2 * \text{backup} - \text{capacity})}{(\text{capacity})} \end{aligned}$$

We calculate demand = (actual demand) by combining Eq. (28) with Eq. (2), (3), and (5) to produce

$$\text{demand} = (\text{design demand}) * \left[1 - \left\{ \begin{aligned} & \left[\begin{aligned} & \left[\begin{aligned} & [\% \text{ decrease (without delay)}] \\ & [\% \text{ decrease (with delay)}] \\ & (\text{speed delay}) \\ & + 30 \frac{[\text{demand} + 2 * \text{backup} - \text{capacity}]}{\text{capacity}} \end{aligned} \right] \\ & \dots \dots \dots (29) \end{aligned} \right\} \right] \end{aligned} \right.$$

which resolves to a direct solution for demand,

$$\text{demand} = \frac{\left[\begin{aligned} & \left\{ (\text{design demand}) * (\text{capacity}) \right\} \\ & * \left\{ \begin{aligned} & \left[1 - [\% \text{ decrease (without delay)}] - [\% \text{ decrease (with delay)}] \right] \\ & * \left[(\text{speed delay}) + 60 * \left(\frac{\text{backup}}{\text{capacity}} \right) - 30 \right] \end{aligned} \right\} \right]}{\text{capacity} + 30 * (\text{design demand}) * [\% \text{ decrease (with delay)}]} \dots \dots \dots (30)$$

We can calculate demand_{9A} by inserting [speed delay (when $D = C$)] = 1.29 min calculated in Sec. 4.3.3, [decrease (with no delay)] = 4.5 % and [decrease (with delay)] = 1.22 %/min calculated in Sec. 6.2, $(\text{design demand})_{9A} = 2,584$ Vph and $\text{capacity}_{9A-1P} = 1,400$ Vph from Table 1, and $(\text{start-of-period backup})_{9A} = (\text{end-of-period backup})_{8A} = 0$ from Table 3 into Eq. (30),

$$(\text{demand})_{9A} = \frac{\left[\frac{\{(2,584) * (1,400)\} * \left\{ \left[1 - [0.045] - [0.0122] \right] * \left[(1.29) + 60 * \left(\frac{0}{1,400} \right) - 30 \right] \right\}}{1,400 + 30 * (2,584) * (0.0122)} \right]}{1,400 + 30 * (2,584) * (0.0122)} = 2,013 \text{ Vph}$$

as shown in Col. 8 of Table 1. From Eq. (28), $(\text{backup delay})_{9A} = 30 * (2,013 + 2 * 0 - 1,400) / (1,400) = 13.14$ min, as shown in Col 2 of Table 3. By Eq. (2), $(\text{work zone delay})_{9A} = 1.29 + 13.14 = 14.43$ min, as shown in Col 5 of Table 3.

Similarly, for period 10A, which has $(\text{start-of-period backup})_{10A} = (\text{end-of-period backup})_{9A} = 613$ V from Table 3,

$$(\text{demand})_{10A} = \frac{\left[\frac{\{(2,176) * (1,400)\} * \left\{ \left[1 - [0.045] - [0.0122] \right] * \left[(1.29) + 60 * \left(\frac{613}{1,400} \right) - 30 \right] \right\}}{1,400 + 30 * (2,176) * (0.0122)} \right]}{1,400 + 30 * (2,176) * (0.0122)} = 1,366 \text{ Vph}$$

as shown in Col. 8 of Table 1. $(\text{backup delay})_{10A} = 30 * (1,366 + 2 * 613 - 1,400) / (1,400) = 25.55$ min, as shown in Col 3 of Table 3. By Eq. (2), $(\text{work zone delay})_{10A} = 1.29 + 25.54 = 26.83$ min, as shown in Col 5 of Table 3.

7.3 CONVERGING ON ACTUAL DEMAND AND DELAY

As stated in Sec. 7.1 above, CO^3 solves iteratively for actual demand and work zone delay when a direct solution is not possible. Fig. 11 shows the iterative steps used to calculate demand and vehicle delay for a period. In summary, a trial demand = $\text{demand}_{\text{trial}}$ is calculated as an estimate of actual demand. A trial delay = $\text{delay}_{\text{trial}}$ is calculated from $\text{demand}_{\text{trial}}$, a new demand = $\text{demand}_{\text{new}}$ is calculated from $\text{delay}_{\text{trial}}$, and a new delay = $\text{delay}_{\text{new}}$ is calculated from $\text{demand}_{\text{new}}$. $\text{Demand}_{\text{new}}$ is then compared to $\text{demand}_{\text{trial}}$, and $\text{delay}_{\text{new}}$ is compared to $\text{delay}_{\text{trial}}$. If either new value differs significantly from its respective trial value, another $\text{demand}_{\text{trial}}$ is calculated, and the steps are repeated until new values are not significantly different from the trial values on which they are based. This section describes the method of calculating trial demands such that the overall iterative process converges efficiently on accurate values of actual demand and vehicle delay.

$\text{Demand}_{\text{trial},1}$ for the first iteration $j = 1$ is calculated based on Eq. (30). $\text{Demand}_{\text{trial},2}$ for the second iteration, $j = 2$, is the average of $\text{demand}_{\text{trial},1}$ and $\text{demand}_{\text{new},1}$, in accord with

$$\text{demand}_{\text{trial},j+1} = \frac{\text{demand}_{\text{trial},j} + \text{demand}_{\text{new},j}}{2} \dots\dots\dots (31)$$

This averaging formula is generally not applied to succeeding iterations, because it does not always converge. High values of [decrease (with delay)] can cause repeated flip-flopping between two alternative sets of trial and new values. Fig. 10 demonstrates such a sensitive case. In Fig. 10 the vertical scale indicates the magnitude of demand, and the arrows indicate that $\text{demand}_{\text{trial},j}$ produced $\text{demand}_{\text{new},j}$, and $\text{demand}_{\text{trial},j-1}$ produced $\text{demand}_{\text{new},j-1}$. When demand and delay are related, Fig. 10 illustrates that the lower the value of $\text{demand}_{\text{trial}}$, the higher the value of $\text{demand}_{\text{new}}$. Therefore, the two arrows cross. To provide dependable and efficient convergence, CO³ sets $\text{demand}_{\text{trial},j+1}$ = the value of $\text{demand}_{\text{trial}}$ with which to start the next iteration, $j+1$, is the value of demand at which the two arrows intersect,

$$\text{demand}_{\text{trial},j+1} = \frac{\text{demand}_{\text{trial},j} * \text{demand}_{\text{new},j-1} - \text{demand}_{\text{trial},j-1} * \text{demand}_{\text{new},j}}{\text{demand}_{\text{trial},j} + \text{demand}_{\text{new},j-1} - \text{demand}_{\text{trial},j-1} - \text{demand}_{\text{new},j}} \dots\dots\dots (32)$$

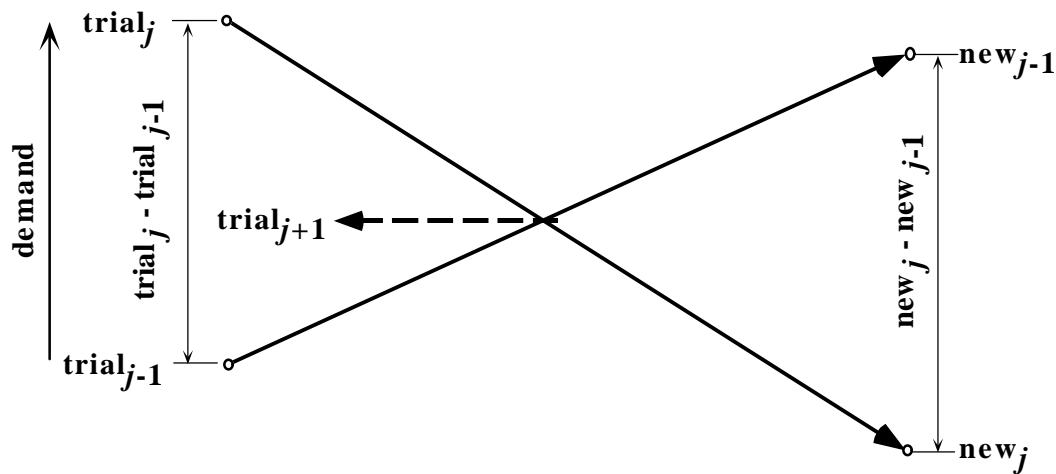


Fig. 10 Calculation of New Trial Actual Demand [change *i* to *j* above]

For each iteration j , we consider a new value is significantly different from its corresponding trial value if its absolute difference exceeds the allowable difference we select,

$$\Delta_{x,j} = |x_{\text{new},j} - x_{\text{trial},j-1}| > \Delta_{x \text{ allowable}} \dots\dots\dots (33)$$

where x = “demand” or “delay”.

To test significance of differences of trial and new values of demand and delay, we input into CO³ values for $\Delta_{\text{demand}\cdot\text{allowable}}$ = maximum allowable difference in demand values and $\Delta_{\text{delay}\cdot\text{allowable}}$ = maximum allowable difference in work zone delay values. For our example problem, we set $\Delta_{\text{demand}\cdot\text{allowable}}$ = (0.1 %) * (maximum design demand) = 0.001 * [(design demand)_{8A} = 3,314 Vph] = 7 Vph and $\Delta_{\text{delay}\cdot\text{allowable}}$ = 0.1 min = 6 sec.

For iteration $j = 1$ for the 12P values of our example, CO³ uses Eq. (30) to calculate $\text{demand}_{\text{trial},1} = 1,315.75$ Vph. From this CO³ calculates $\text{delay}_{\text{trial},1} = 0.17643$ hr = 10.586 min. From this CO³ calculates $\text{demand}_{\text{new},1} = 1,325.88$ Vph, from which $\text{delay}_{\text{new},1} = 0.17936$ hr = 10.762 min. The absolute values of differences between trial and new values of demand and delay are calculated using Eq. (33), $\Delta_{\text{demand},1} = |1,315.75 - 1,325.88| = 10.13$ Vph >

$\Delta_{\text{demand-allowable}} = 7 \text{ Vph}$ and $\Delta_{\text{delay},1} = |10.586 - 10.762| = 0.176 \text{ min} > \Delta_{\text{delay-allowable}} = 0.1 \text{ min}$. We must perform another iteration, because $\Delta_{\text{demand},1}$ exceeds $\Delta_{\text{demand-allowable}}$ and also because $\Delta_{\text{delay},1}$ exceeds $\Delta_{\text{delay-allowable}}$.

We use Eq. (31) for iteration $j = 2$ to calculate $\text{demand}_{\text{trial},2} = (1,315.75 + 1,325.88) / 2 = 1,320.81 \text{ Vph}$. From this CO^3 calculates $\text{delay}_{\text{trial},2} = 0.17789 \text{ hr} = 10.763 \text{ min}$, from which $\text{demand}_{\text{new},2} = 1,324.17 \text{ Vph}$, from which $\text{delay}_{\text{new},2} = 0.17886 \text{ hr} = 10.732 \text{ min}$. The absolute values of differences between trial and new values of demand and delay are $\Delta_{\text{demand},2} = |1,320.81 - 1,324.17| = 3.36 \text{ Vph} < \Delta_{\text{demand-allowable}} = 7 \text{ Vph}$ and $\Delta_{\text{delay},2} = |10.763 - 10.732| = 0.031 \text{ min} < \Delta_{\text{delay-allowable}} = 0.1 \text{ min}$. We need not perform another iteration, because differences between trial and new values of both demand and delay are within their allowables and are therefore not significant. If either $\Delta_{\text{demand},2}$ or $\Delta_{\text{delay},2}$ exceeded its allowable, CO^3 would have used Eq. (32) to calculate $\text{demand}_{\text{trial},3}$,

$$\text{demand}_{\text{trial},3} = \frac{1,320.81 * 1,325.88 - 1,315.75 * 1,324.17}{1,320.81 + 1,325.88 - 1,315.75 - 1,324.17} = 1,323.32 \text{ Vph}$$

For its output, CO^3 uses the latest value calculated. Therefore, $\text{delay}_{12\text{P}} = \text{delay}_{\text{new},2} = 0.17886 \text{ hr} = 10.73 \text{ min}$ as shown in Col. 5 of Table 3. From this CO^3 calculates $\text{demand}_{12\text{P}} = 1,323 \text{ Vph}$, as shown in Col. 8 of Table 1.

7.4 OVERALL VIEW OF CONVERGENCE

As stated above, demand decrease, actual demand, and delay cannot always be calculated directly, because demand decrease is dependent on average delay per vehicle, and average delay per vehicle is dependent on actual demand, which is dependent on demand decrease. CO^3 uses the following iterative method where necessary to calculate these variables for each period, which is diagrammed in Fig. 11.

1. Estimate trial value of actual demand = $\text{demand}_{\text{trial}}$ for period i , based on input data:
2. Calculate trial values of delay: $(\text{backup delay})_{\text{trial}}$ and $(\text{speed delay})_{\text{trial}}$, based on $\text{demand}_{\text{trial}}$ for i :
3. Calculate a trial average delay per vehicle = $\text{delay}_{\text{trial}} = (\text{backup delay})_{\text{trial}}$ and $(\text{speed delay})_{\text{trial}}$.
4. Calculate new estimate of decrease in demand = $\text{decrease}_{\text{new}}$ based on $\text{delay}_{\text{trial}}$.
5. Calculate new estimate of actual demand = $\text{demand}_{\text{new}}$ based on $\text{decrease}_{\text{new}}$.
6. Calculate new estimates of delay: $(\text{backup delay})_{\text{new}}$ and $(\text{speed delay})_{\text{new}}$, based on $\text{demand}_{\text{new}}$.
7. Calculate new estimate of average delay per vehicle = $\text{delay}_{\text{new}} = (\text{backup delay})_{\text{new}}$ and $(\text{speed delay})_{\text{new}}$.
8. Determine whether the new estimates of demand or delay are significantly different from the trial estimates. I.e., is $\text{demand}_{\text{new}}$ significantly different from $\text{demand}_{\text{trial}}$ or is $\text{delay}_{\text{new}}$ significantly different from $\text{delay}_{\text{trial}}$?

- If either new estimate is significantly different, calculate a trial value of demand = demand_{trial} based on the current and immediate-past values of demand_{trial} and demand_{new}, and repeat steps 1-8 for the new demand_{trial}.
- If neither new estimate is significantly different, then set the values of actual demand, average delay, and other demand and delay variables for period *i* to the new estimated values, and perform steps 1-8 above for time period (*i* + 1) = period following *i*.

8 USER COST OF DEMAND DECREASES

8.1 GENERAL

Decreases in design demand consist of cancellations and diversions of cars and trucks. User cost per vehicle of decrease is

$$\text{decrease user cost (\$)} = \left[\text{user cost per decreased vehicle (\$/V)} \right] \dots \dots \dots (34) \\ * \left[\text{demand decrease (V)} \right]$$

8.2 USER COST OF CANCELLATIONS

User cost per cancellation is input, as shown in Fig. 5 (a). For example, in Fig. 5 (a) (user cost per car cancellation) = \$4.00 /V and (user cost per truck cancellation) = \$10.00 / V. Inputting canceled car values from Fig. 8 into Eq. (4) for period 9A, (canceled cars)_{9A} = [(2 %) + (0.3 %/min) * (14.43 min)] * [(90 %) * (2,584)] = [6.33 %] * [2,326] = 147 V. From Eq. (34), (canceled car user cost)_{9A} = (\$4.00 / V) * (147 V) = \$588, (canceled truck user cost)_{9A} = 0, because Fig. 8 shows that no trucks cancel due to delay.

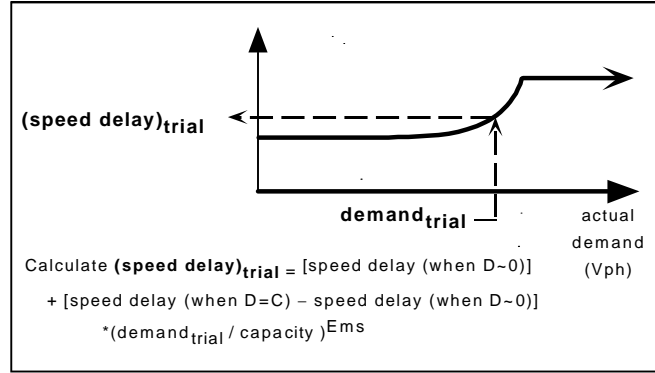
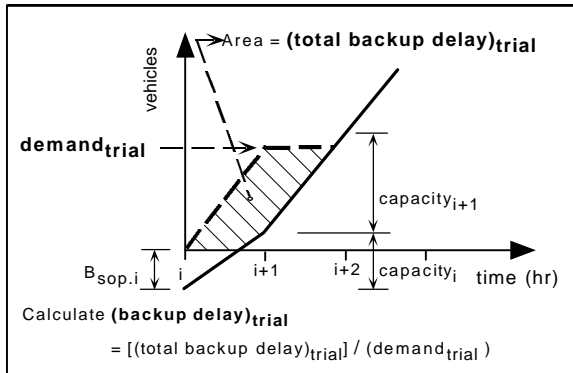
8.3 USER COST OF DIVERSIONS

For calculating diversion user cost, diversion travel times are calculated by Eq. (6), into which we input values from Fig. 5 (a),

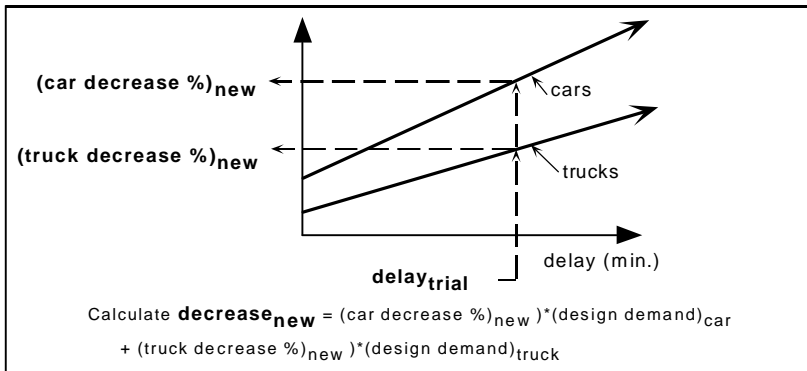
$$\text{diversion normal time} = \frac{\text{diversion normal distance}}{\text{diversion normal speed}} \dots \dots \dots (35) \\ = \frac{4 \text{ mi}}{70 \text{ mph}} = 0.05714 \text{ hr} = 3.43 \text{ min.}$$

$$\text{diversion method time} = \frac{\text{diversion method distance}}{\text{diversion method speed}} \dots \dots \dots (36) \\ = \frac{10 \text{ mi}}{45 \text{ mph}} = 0.22222 \text{ hr} = 13.33 \text{ min.}$$

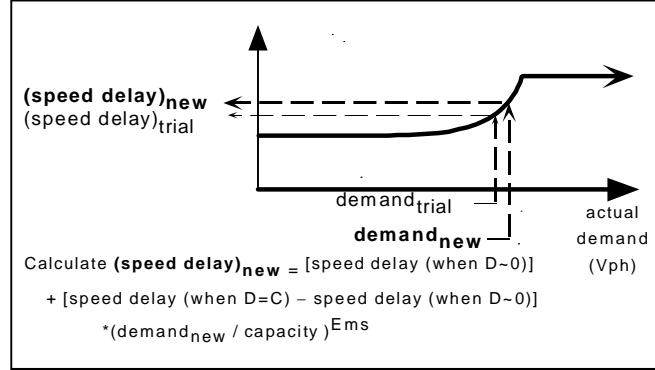
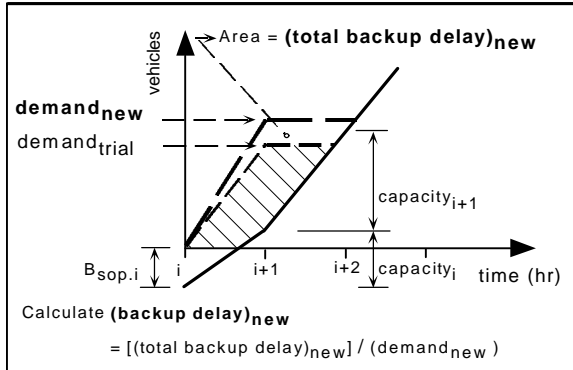
For period i : Estimate $\text{demand}_{\text{trial}}$



Calculate **delay_{trial}** = **(backup delay)_{trial}** + **(speed delay)_{trial}**



Calculate **demand_{new}** = **(design demand)** * **(1 - decrease_{new})**



Calculate **delay_{new}** = **(backup delay)_{new}** + **(speed delay)_{new}**

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Test for Change: If $[\text{abs}(\text{delay}_{\text{new}} - \text{delay}_{\text{trial}}) \geq e_{\text{del}}]$ or $[\text{abs}(\text{demand}_{\text{new}} - \text{demand}_{\text{trial}}) \geq e_{\text{D}}]$
 then **estimate** new $\text{demand}_{\text{trial}}$ based on old $\text{demand}_{\text{trial}}$ and old $\text{demand}_{\text{new}}$, and **Recycle** for i ,
 else **set** **(actual demand)** = $\text{demand}_{\text{new}}$, **delay** = $\text{delay}_{\text{new}}$, **Do calculations** for period $i+1$

Fig. 11 Steps in Calculating Actual Demand and Delay for Period i .

By Eq. (9), (diversion delay) = 13.33 – 3.43 = 9.90 min, as shown in Fig. 5 (a). User cost per diversion is calculated using Eq. (15). For example,

$$\begin{aligned} \text{car diversion user cost} &= \frac{9.90 \text{ min}}{60 \text{ min / hr}} * (\$12.00 / \text{hr}) + (10 \text{ mi} - 4 \text{ mi}) * (\$0.30 / \text{mi}) \\ &= 0.165 * (12.00) + 6 * (0.30) = \$1.98 + \$1.80 = \$3.78 / \text{V} \end{aligned}$$

as shown in Fig. 5 (a). Similarly, truck diversion cost uses the same (diversion delay) = 9.90 min and distances, but different costs per hour and per mile to produce (truck diversion user cost) = 0.165 * (30.00) + 6 * (1.00) = 4.95 + 6.00 = \$10.95 / V.

Inputting diverted car values from Fig. 8 into Eq. (4) for period 9A, (diverted cars)_{9A} = [(3 %) + (1 %/min) * (14.43 min)] * [(90 %) * (2,584)] = [17.43 %] * [2,326] = 405 V. From Eq. (34), (diverted car user cost)_{9A} = (\$3.78 / V) * (405 V) = \$1,531. Also, (diverted trucks)_{9A} = [(0 %) + (0.5 %/min) * (14.43 min)] * [(10 %) * (2,584)] = [7.22 %] * [258] = 19 V, and (diverted truck user cost)_{9A} = (\$10.95 / V) * (19 V) = \$208. Adding these costs produces

$$\text{decrease cost} = \text{canceled costs} + \text{diversion cost} \dots\dots\dots (37)$$

and (decrease cost)_{9A} = 588 + 0 + 1,531 + 208 = \$2,327 ≅ \$2,325 as shown in Col. 8 of Table 3, within round-off errors.

9 SUMMARY AND CONCLUSIONS

Design demand is the normal demand when there is no work zone. Capacity of a work zone is less than normal, and maximum speed in a work zone is less than normal. Demand is closer to work zone capacity than normal, because capacity is less than normal, which reduces work zone speed further. Reduced work zone speed produces speed delay. Backup occurs in the amount by which work zone demand exceeds capacity. Backup delay occurs as vehicles in the backup queue wait to enter the work zone. Work zone delay is the sum of speed delay and backup delay.

When work zone delay causes traffic to divert or cancel trips, actual demand through the work zone becomes less than design demand. Traffic that diverts has its own delay due to the additional distance and lower speeds of alternate routes. Work zone delay, diverted and cancelled traffic, and work zone demand share a circular dependency.

As CO³ users, we estimate traffic distances and speeds under normal and work zone conditions and along alternate routes. We also estimate the dependency between work zone delay and the fractions of car and truck traffic that divert and cancel trips. From this input, CO³ calculates delays, diversions, cancellations, and actual demand for each period. CO³ is unique in resolving the interdependency between work zone delay and work zone demand.

Based on input of car and truck user cost per hour and mile of travel and per cancellation, CO³ calculates road user cost for work zone demand, diversions, and cancellations. For each period CO³ provides detailed output of car and truck diversions, cancellations, and work zone demand; backup, backup delay, and speed delay; and user costs of delay and traffic decreases. Output provides a detailed view of construction impact for each period and each day. We can input alternative lane closure patterns and calculate and compare the impact of alternative traffic maintenance plans. By integrating traffic impacts and user costs with construction costs of alternative plans (not shown here), we can select a preferred method of performing construction and maintaining traffic that is an acceptable balance of public cost and benefit.

APPENDIX 1 REFERENCES

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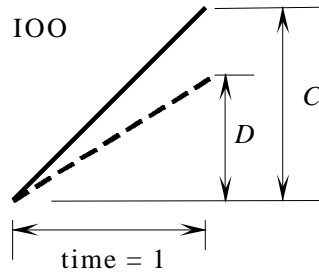
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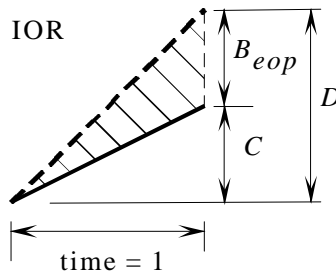
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APPENDIX 2 PERIOD DELAY EQUATIONS

A.1 Class IOO: $C > D; B_{sop} = 0$



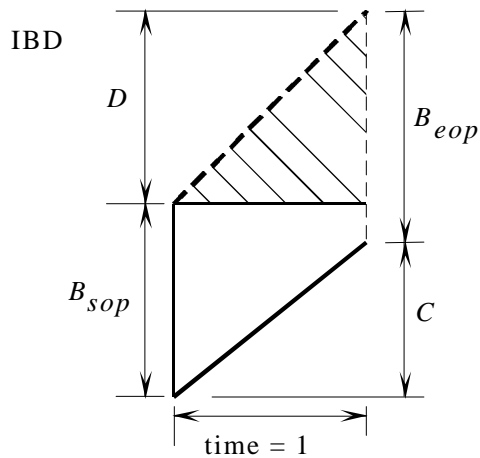
A.2 Class IOR: $D > C; B_{sop} = 0$



$$(\text{backup delay})_i = \frac{D-C}{2} \dots \dots \dots (38)$$

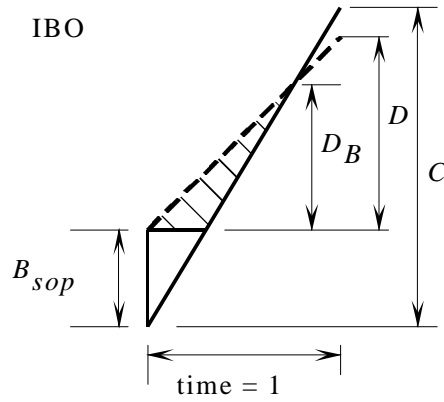
$$(\text{speed delay})_i = C * [\text{speed delay } (D = C)] \dots \dots \dots (39)$$

A.3 Class IBD: $B_{sop} + D - C; \leq D$



$$(\text{backup delay})_i = \frac{D}{2} \dots \dots \dots (40)$$

A.4 Class IBO: $B_{sop} + D - C < 0$

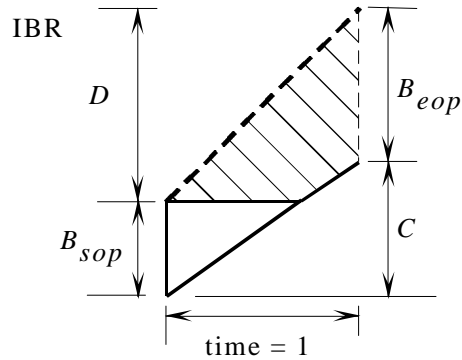


$$D_B = \frac{D * B_{sop}}{C - D}$$

$$\text{total backup delay} = \frac{D_B * B_{sop}}{2} * \frac{B_{sop}}{C} = \frac{D * B_{sop}^2}{2C(C - D)} \dots \dots \dots (41)$$

$$\begin{aligned} \text{total speed delay} &= D_B * [\text{speed delay } (D = C)] \\ &+ (D - D_B) * [\text{speed delay } (0 < D < C)] \dots \dots \dots (42) \end{aligned}$$

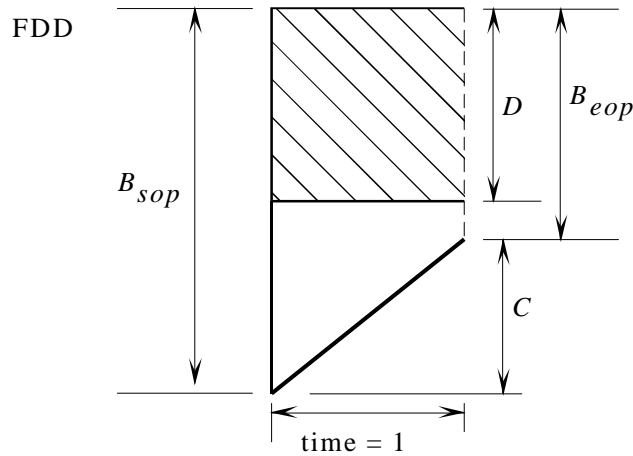
A.5 Class IBR: $0 < (B_{sop} + D - C) < D$



$$(\text{backup delay})_i = \frac{1}{2} \left[D - \frac{(C - B_{sop})^2}{C} \right] \dots \dots \dots (43)$$

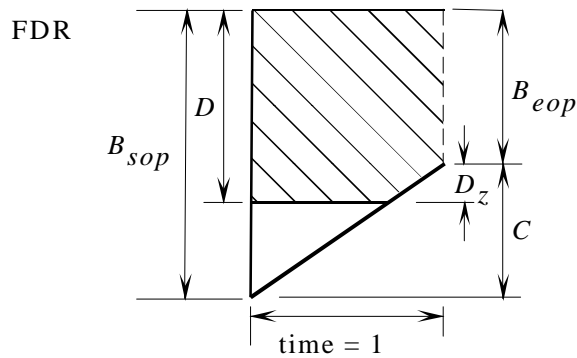
$$(\text{speed delay})_i = (C - B_{sop}) * [\text{speed delay } (D = C)] \dots \dots \dots (44)$$

A.6 Class FDD: $(B_{sop} - C) \geq D$



$$(\text{backup delay})_i = D \dots \dots \dots (45)$$

A.7 Class FDR: $0 < (B_{sop} - C) < D \leq B_{sop}$

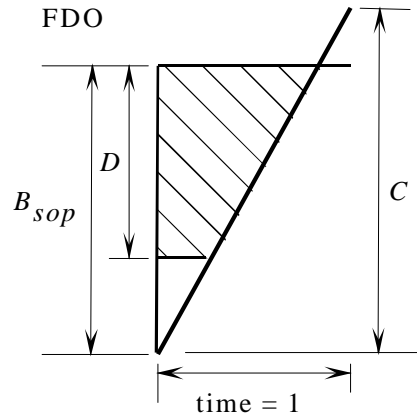


$$D_z = D + C - B_{sop} \dots \dots \dots (46)$$

$$(\text{backup delay})_i = D - \frac{D_z^2}{2 * C} \dots \dots \dots (47)$$

$$(\text{speed delay})_i = D_z * [\text{speed delay } (D = C)] \dots \dots \dots (48)$$

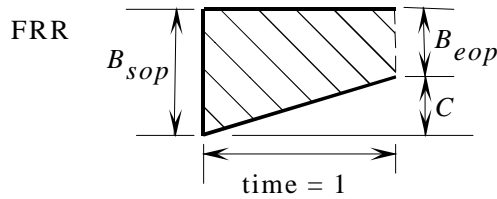
A.8 Class FDO: $D \leq B_{sop} \leq C$



$$(\text{backup delay})_i = \frac{D}{C} * \left(B_{sop} - \frac{D}{2} \right) \dots \dots \dots (49)$$

$$(\text{speed delay})_i = D * [\text{speed delay } (D = C)] \dots \dots \dots (50)$$

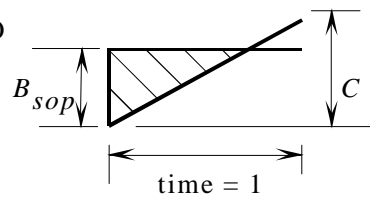
A.9 Class FRR: $C < B_{sop} < D$



$$(\text{backup delay})_i = B_{sop} - \frac{C}{2} \dots \dots \dots (51)$$

$$(\text{speed delay})_i = C * [\text{speed delay } (D = C)] \dots \dots \dots (52)$$

A.10 Class FRO: $C < B_{sop} < D$



$$(\text{backup delay})_i = \frac{B_{sop}^2}{2 * C} \dots \dots \dots (53)$$

$$(\text{speed delay})_i = B_{sop} * [\text{speed delay } (D = C)] \dots \dots \dots (54)$$